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Supersymmetric string vacua on $AdS_3 \times \mathcal{N}$

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Abstract: String backgrounds of the form $AdS_3 \times \mathcal{N}$ that give rise to two dimensional spacetime superconformal symmetry are constructed.

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1. Introduction

In this letter we study the conditions on curved string backgrounds of the form $AdS_3 \times \mathcal{N}$ that give rise to spacetime superconformal symmetry. We use the NSR formulation; for simplicity we describe the left moving sector only — it can be combined with the right-moving sector in the standard way. String theories on $AdS_3 \times \mathcal{N}$ were studied for the bosonic case in [1, 2, 3]. Examples of supersymmetric strings in the NSR formulation were studied in, e.g. [1, 4, 5, 6, 3, 7]. For some early work on string theory on AdS_3 see e.g. [8].

The main results of this work are the following: If $\mathcal{N}$ has an affine U(1) symmetry and $\mathcal{N}$/U(1) has an $N = 2$ worldsheet superconformal symmetry, then there is a construction of a superstring with two-dimensional $N = 2$ *spacetime* superconformal symmetry. A Z_2 quotient of this construction leads to a family of theories with two-dimensional $N = 1$ *spacetime* superconformal symmetry. We also discuss conditions for $N > 2$ superconformal symmetry: These involve an $\mathcal{N}$ with an SU(2) factor whose level is determined in terms of the level of the AdS_3 background.

This investigation is the analog of the study of supersymmetric backgrounds for compactification to Minkowski space $\mathcal{M}_d$ in $d = 3$ or 4 dimensions [9, 10, 11, 12]. String theories on $\mathcal{M}_4 \times \mathcal{N}$ have four-dimensional $N = 1$ spacetime supersymmetry provided $\mathcal{N}$ has an $N = 2$ worldsheet superconformal symmetry [9, 10]. String theories on $\mathcal{M}_3 \times \mathcal{N}$ have three-dimensional $N = 2$ spacetime supersymmetry if $\mathcal{N}$ has an affine U(1) and $\mathcal{N}$/U(1) has an $N = 2$ worldsheet superconformal symmetry. A Z_2 quotient of such an $\mathcal{N}$ leaves a three-dimensional $N = 1$ spacetime supersymmetry; in this case, the symmetry algebra on $\mathcal{N}$ can be extended to a nonlinear algebra associated to manifolds with G_2 holonomy [11, 12].

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The structure of the paper is as follows: In section 2, we describe the worldsheet properties that lead to spacetime supersymmetry on $AdS_3 \times \mathcal{N}$. In section 3, we construct the two dimensional $N = 2$ spacetime superconformal algebra associated with the boundary CFT of AdS_3. In section 4, we take a quotient of the $N = 2$ construction to find a class of models with $N = 1$ spacetime superconformal symmetry. In section 5, we discuss models with $N > 2$ spacetime superconformal symmetry. Finally, in section 6, we comment on new examples that arise from our results and other issues.

2. Worldsheet properties of fermionic strings on $AdS_3 \times \mathcal{N}$

We first consider the AdS_3 factor of the background. This theory has affine SL(2) currents

$$\psi^A + \theta\sqrt{\frac{2}{k}}J^A\,, \qquad A = 1, 2, 3, \tag{2.1}$$

where

$$J^A = j^A - \frac{i}{2}\epsilon^A{}_{BC}\psi^B\psi^C\,, \tag{2.2}$$

and

$$\begin{aligned} \psi^A(z)\psi^B(w) &\sim \frac{\eta^{AB}}{z-w}\,, \qquad \eta^{AB} = \mathrm{diag}(+,+,-)\,, \\ J^A(z)J^B(w) &\sim \frac{\frac{k}{2}\eta^{AB}}{(z-w)^2} + \frac{i\epsilon^{AB}{}_C J^C}{z-w}\,. \end{aligned} \tag{2.3}$$

The purely bosonic currents j^A generate an affine SL(2) algebra at level $k+2$ and commute with ψ^A, whereas the total currents J^A generate a level k SL(2) algebra and act on ψ as follows from (2.2), (2.3). The central charge of the AdS_3 part of the theory is thus

$$c^{\mathrm{SL}(2)} = \frac{3(k+2)}{k} + \frac{3}{2}\,, \tag{2.4}$$

where the two terms are the bosonic and fermionic contributions, respectively. The $N = 1$ worldsheet supercurrent is

$$T_F^{\mathrm{SL}(2)} = \sqrt{\frac{2}{k}}\left(\psi^A j_A - i\psi^1\psi^2\psi^3\right)\,. \tag{2.5}$$

The internal space $\mathcal{N}$ is described by a unitary superconformal field theory (CFT) background with central charge

$$c^{\mathcal{N}} = 15 - c^{\mathrm{SL}(2)} = \frac{21}{2} - \frac{6}{k}\,. \tag{2.6}$$

We denote the worldsheet supercurrent of $\mathcal{N}$ by $T_F^{\mathcal{N}}$.

The construction of $N = 2$ spacetime supersymmetry described in section 3 below requires that $\mathcal{N}$ have an affine U(1) symmetry with an $N = 1$ current

$$\psi^{\mathrm{U}(1)} + \theta J^{\mathrm{U}(1)}, \tag{2.7}$$

where

$$\begin{aligned} \psi^{\mathrm{U}(1)}(z)\psi^{\mathrm{U}(1)}(w) &\sim \frac{1}{z-w}, \\ J^{\mathrm{U}(1)}(z)J^{\mathrm{U}(1)}(w) &\sim \frac{1}{(z-w)^2}, \\ J^{\mathrm{U}(1)}(z)\psi^{\mathrm{U}(1)}(w) &\sim 0, \end{aligned} \tag{2.8}$$

and a worldsheet supercurrent

$$T_F^{\mathrm{U}(1)} = \psi^{\mathrm{U}(1)} J^{\mathrm{U}(1)}. \tag{2.9}$$

It is convenient to bosonize the affine current

$$J^{\mathrm{U}(1)} = i\partial Y, \tag{2.10}$$

where Y is a canonically normalized scalar: $Y(z)Y(w) \sim -\log(z-w)$.

We can construct the quotient CFT $\mathcal{N}/\mathrm{U}(1)$ with the supercurrent

$$T_F^{\mathcal{N}/\mathrm{U}(1)} = T_F^{\mathcal{N}} - T_F^{\mathrm{U}(1)}; \tag{2.11}$$

this has a central charge

$$c^{\mathcal{N}/\mathrm{U}(1)} = c^{\mathcal{N}} - c^{\mathrm{U}(1)} = 9 - \frac{6}{k}. \tag{2.12}$$

The construction of $N = 2$ spacetime supersymmetry described in section 3 below further requires that $\mathcal{N}/\mathrm{U}(1)$ have an $N = 2$ superconformal algebra (which commutes with the U(1) above). In particular, its $\mathrm{U}(1)_R$-current $J_R^{\mathcal{N}/\mathrm{U}(1)}$ has the standard normalization

$$J_R^{\mathcal{N}/\mathrm{U}(1)}(z) J_R^{\mathcal{N}/\mathrm{U}(1)}(w) \sim \frac{1}{3}\frac{c^{\mathcal{N}/\mathrm{U}(1)}}{(z-w)^2}. \tag{2.13}$$

We bosonize $J_R^{\mathcal{N}/\mathrm{U}(1)}$ in terms of a canonically normalized scalar Z by

$$J_R^{\mathcal{N}/\mathrm{U}(1)} = i\sqrt{\frac{c^{\mathcal{N}/\mathrm{U}(1)}}{3}}\partial Z \equiv ia\partial Z, \tag{2.14}$$

where

$$a \equiv \sqrt{3 - \frac{2}{k}}. \tag{2.15}$$

The worldsheet supercurrent $T_F^{\mathcal{N}/\mathrm{U}(1)}$ can be decomposed into two parts with R-charges ± 1; these charges can be expressed in terms of explicit Z dependent factors to give:

$$T_F^{\mathcal{N}/\mathrm{U}(1)} = e^{\frac{i}{a}Z}\tau_+ + e^{-\frac{i}{a}Z}\tau_- \,, \tag{2.16}$$

where $\tau_\pm$ carry *no* R-charge, i.e.

$$\begin{aligned} J_R^{\mathcal{N}/\mathrm{U}(1)}(z)e^{\pm\frac{i}{a}Z}(w) &\sim \frac{\pm e^{\pm\frac{i}{a}Z}}{z-w} \\ J_R^{\mathcal{N}/\mathrm{U}(1)}(z)\tau_\pm(w) &\sim 0\,. \end{aligned} \tag{2.17}$$

3. $N = 2$ spacetime superconformal theories

We now construct an $N = 2$ superconformal algebra in spacetime out of the worldsheet ingredients described above. As in [1], we introduce canonically normalized scalars H_I with $I = 0, 1, 2$:

$$\begin{aligned} \partial H_1 &= \psi^1\psi^2 \\ i\partial H_2 &= \psi^3\psi^{\mathrm{U}(1)} \\ i\sqrt{3}\partial H_0 &= J_R^{\mathcal{N}/\mathrm{U}(1)} - \sqrt{\frac{2}{k}}J^{\mathrm{U}(1)}\,, \end{aligned} \tag{3.1}$$

where

$$H_I(z)H_J(w) \sim -\delta_{IJ}\log(z-w)\,. \tag{3.2}$$

For future reference we remind the reader that

$$e^{\pm iH_2} = \frac{i}{\sqrt{2}}(\psi^3 \pm \psi^{\mathrm{U}(1)})\,. \tag{3.3}$$

The *spacetime* supercharges are constructed as [13]

$$\mathbf{G}_r^\pm = (2k)^{\frac{1}{4}}\oint dz\, e^{-\frac{\phi}{2}}S_r^\pm\,, \qquad r = \pm\frac{1}{2}\,, \tag{3.4}$$

where ϕ is the scalar field arising in the bosonized β, γ superghost system of the worldsheet supersymmetry, and the spin fields $S_r^\pm$ are (recall (3.1), (2.15), (2.14) and (2.10))

$$\begin{aligned} S_r^+ &= e^{ir(H_1+H_2)+i\frac{\sqrt{3}}{2}H_0} = e^{ir(H_1+H_2)+i\frac{a}{2}Z-i\sqrt{\frac{1}{2k}}Y}\,, \\ S_r^- &= e^{ir(H_1-H_2)-i\frac{\sqrt{3}}{2}H_0} = e^{ir(H_1-H_2)-i\frac{a}{2}Z+i\sqrt{\frac{1}{2k}}Y}\,. \end{aligned} \tag{3.5}$$

(We neglect the usual cocycle factors). The supercharges $\mathbf{G}_r^\pm$ are physical only if they are BRST invariant. This requires that the OPE of $T_F(z)S_r^\pm(w)$ have no $(z-w)^{-3/2}$ singularity. Here T_F is the total worldsheet $N = 1$ supercurrent:

$$T_F = T_F^{\mathrm{SL}(2)} + T_F^{\mathrm{U}(1)} + T_F^{\mathcal{N}/\mathrm{U}(1)} \tag{3.6}$$

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(see (2.5), (2.9), (2.11)). Consider

$$S_{\epsilon_1\epsilon_2\epsilon} = e^{\frac{i}{2}\left(\epsilon_1 H_1 + \epsilon_2 H_2 + \epsilon\left(aZ - \sqrt{\frac{2}{k}}Y\right)\right)}, \qquad \epsilon_1, \epsilon_2, \epsilon = \pm 1\,; \tag{3.7}$$

from (2.16) we find

$$T_F^{\mathcal{N}/\mathrm{U}(1)}(z) S_{\epsilon_1\epsilon_2\epsilon}(w) \sim (z-w)^{\frac{\epsilon}{2}}(\cdots)\tau_+ + (z-w)^{-\frac{\epsilon}{2}}(\cdots)\tau_-\,, \tag{3.8}$$

where $(\cdots)$ represents irrelevant factors. Therefore $T_F^{\mathcal{N}/\mathrm{U}(1)}(z)S_r^{\pm}(w)$ has *no* $(z-w)^{-3/2}$ singularity, and the only possible sources of such singularities are $T_F^{\mathrm{U}(1)}$ and the $\psi^1\psi^2\psi^3$ term in $T_F^{\mathrm{SL}(2)}$. These two contributions cancel each other for $\epsilon_1\epsilon_2\epsilon = 1$, as can be seen using

$$\psi^{\mathrm{U}(1)} J^{\mathrm{U}(1)} - i\sqrt{\frac{2}{k}}\psi^1\psi^2\psi^3 = \left(\frac{1}{\sqrt{2}}\partial Y - \frac{1}{\sqrt{k}}\partial H_1\right)e^{iH_2} - \left(\frac{1}{\sqrt{2}}\partial Y + \frac{1}{\sqrt{k}}\partial H_1\right)e^{-iH_2} \tag{3.9}$$

(see (2.10), (3.1), (3.3)). Substituting all 4 solutions of $\epsilon_1\epsilon_2\epsilon = 1$ into (3.7), we recover (3.5). In addition, $e^{-\phi/2}S_r^{\pm}$ are mutually local. This completes the proof that $\mathbf{G}_r^{\pm}$ as defined in (3.4) are physical.

The algebra generated by the supercharges is

$$\begin{aligned}
\{\mathbf{G}_r^+, \mathbf{G}_s^-\} &= 2\mathbf{L}_{r+s} + (r-s)\mathbf{J}_0\,, \qquad r,s = \pm\frac{1}{2}\\
[\mathbf{L}_m, \mathbf{L}_n] &= (m-n)\mathbf{L}_{m+n}\,, \qquad m,n = 0, \pm 1\\
[\mathbf{L}_m, \mathbf{G}_r^{\pm}] &= \left(\frac{m}{2} - r\right)\mathbf{G}_{m+r}^{\pm}\\
[\mathbf{J}_0, \mathbf{G}_r^{\pm}] &= \pm\mathbf{G}_r^{\pm}
\end{aligned} \tag{3.10}$$

with all other (anti)commutators vanishing. Up to picture-changing [13], $\mathbf{L}_0, \mathbf{L}_{\pm1}, \mathbf{J}_0$ are given by (recall (2.1), (2.7))

$$\begin{aligned}
\mathbf{L}_0 &= -\oint J^3\,,\\
\mathbf{L}_{\pm1} &= -\oint \frac{1}{\sqrt{2}}(J^1 \pm iJ^2)\,,\\
\mathbf{J}_0 &= -\sqrt{2k}\oint J^{\mathrm{U}(1)}\,.
\end{aligned} \tag{3.11}$$

The algebra (3.10) is a global spacetime $N=2$ superconformal algebra.

String theory on $AdS_3 \times \mathcal{N}$ has a *full* spacetime Virasoro symmetry $\mathbf{L}_n$, with $n \in \mathbb{Z}$ [1, 3]; commuting $\mathbf{L}_n$ with the generators of the global algebra (3.10) gives a full spacetime $N=2$ superconformal algebra in the spacetime NS-sector with modes $\mathbf{G}_r^{\pm}$, $r \in \mathbb{Z}+1/2$ and $\mathbf{J}_n$, $n \in \mathbb{Z}$. Physical states are constructed using physical vertex operators that are local with respect to the supercharges (3.4); this is the analog of the usual *GSO* projection.[1]

[1] Strictly speaking, the full Virasoro algebra and physical states were constructed in the Euclidean version of AdS_3 [1, 3]. The construction of the finite Lie superalgebra (3.10) uses only the algebraic structure of SL(2), and is independent of the representation theory; hence it is also valid for the Lorentzian case.

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4. $N = 1$ spacetime superconformal theories

The construction of the previous section gave us two dimensional $N = 2$ spacetime superconformal symmetry. It is straightforward to find a Z_2 quotient that preserves exactly half of the spin fields (3.5) and leads to $N = 1$ spacetime superconformal symmetry. This quotient is analogous to the construction of manifolds with G_2 holonomy by a Z_2 quotient of a product of a Calabi-Yau manifold with an S^1 [12].

Concretely, we break the $N = 2$ superconformal symmetry of $\mathcal{N}/\mathrm{U}(1)$ by the quotient with respect to $J_R^{\mathcal{N}/\mathrm{U}(1)} \to -J_R^{\mathcal{N}/\mathrm{U}(1)}$; simultaneously, we take the quotient with respect to $J^{\mathrm{U}(1)} \to -J^{\mathrm{U}(1)}$ and $\psi^{\mathrm{U}(1)} \to -\psi^{\mathrm{U}(1)}$. This has the net effect of identifying $\{H_1, H_2, H_0\} \to \{H_1, -H_2, -H_0\}$ (see (3.1)), and thus $S_r^\pm \to S_r^\mp$ (3.5). Therefore, the spacetime superconformal symmetry is projected to the $N = 1$ subalgebra generated by the symmetric combination $\mathbf{G}_r^+ + \mathbf{G}_r^-$.

This indeed resembles the construction of superconformal models on manifolds with G_2 holonomy [11, 12], except that the total central charge of $\mathcal{N}$ is not 21/2 but $21/2-6/k$ (see (2.6)). It would be interesting to see if one can find a general nonlinear worldsheet algebra that characterizes $\mathcal{N}$ and then use the methods of [11] to generate the $N = 1$ spacetime superconformal symmetry in the general $AdS_3 \times \mathcal{N}$ case.

5. $N > 2$ spacetime superconformal theories

We may also consider the extension of the methods of section 3 to models with $N > 2$ symmetry. This gives rise to models that have been considered on a case by case basis in the literature [1, 4, 5, 6].

The small $N = 4$ superconformal algebra (see [1] and references therein) has an affine SU(2) R-symmetry. As explained in [1], this spacetime affine SU(2) arises from a level k worldsheet affine SU(2) factor in $\mathcal{N}$. For the construction of section 3, we may take $J^{\mathrm{U}(1)}$ as the Cartan generator of $\mathrm{SU}(2)_k$. The remaining background $\mathcal{N}_{(c=6)} \equiv \mathcal{N}/\mathrm{SU}(2)_k$ is precisely a $c = 6$ CFT, and small $N = 4$ spacetime supersymmetry requires that $\mathcal{N}_{(c=6)}$ have small $N = 4$ worldsheet supersymmetry. This can be shown by the methods in [10], where it is argued that for compactification to four dimensional Minkowski space $\mathcal{M}_4 \times \mathcal{N}_{(c=9)}$, $N = 2$ spacetime supersymmetry on $\mathcal{M}_4$ requires that $\mathcal{N}_{(c=9)}$ factorize as $\mathcal{N}_{(c=9)} = \mathcal{N}_{(c=6)} \times T^2$ with small $N = 4$ worldsheet superconformal symmetry on $\mathcal{N}_{(c=6)}$.

The large $N = 4$ superconformal algebra (see [4] and references therein) has an affine $\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ R-symmetry. Again, as explained in [1, 4], this spacetime affine algebra arises from a worldsheet algebra $\mathrm{SU}(2)_{k'} \times \mathrm{SU}(2)_{k''} \times \mathrm{U}(1)$ where the levels are related to the level k of the AdS_3 factor by $1/k = 1/k' + 1/k''$. This implies that the central charge of the $\mathrm{SU}(2)_{k'} \times \mathrm{SU}(2)_{k''} \times \mathrm{U}(1)$ factor is $c = 21/2 - 6/k$, and so completely determines $\mathcal{N}$ (2.6). For the construction of section 3, we may take $J^{\mathrm{U}(1)}$ as the diagonal Cartan generator of $\mathrm{SU}(2)_{k'} \times \mathrm{SU}(2)_{k''}$; this implies that H_2 of (3.1) above is $-H_4$ of equation (2.31) in [4].

To construct $N = 3$ spacetime superconformal models, we may for instance take a Z_2 quotient of the large $N = 4$ model in such a way as to preserve 3 out of 4 spacetime supersymmetries. This is worked out in detail in [6]; the basic idea is to take the construction of [4] with $k' = k''$ and quotient by a Z_2 action that exchanges the two SU(2) factors in $\mathcal{N}$ and simultaneously reflects the U(1) factor in $\mathcal{N}$. Since the $J^{\mathrm{U}(1)}$ current we use is in the diagonal of $\mathrm{SU}(2) \times \mathrm{SU}(2)$ and hence inert under this quotient, the construction survives and gives an $N = 2$ subalgebra of the $N = 3$ spacetime superconformal algebra discussed in [6].

In models with enhanced spacetime superconformal symmetries, one has to take some care in choosing $J^{\mathrm{U}(1)}$, as an arbitrary choice may lead to supercharges that are not mutually local with the spacetime R-symmetries, and thus preserve only the $N = 2$ subalgebra (3.10).

6. Examples and discussion

We close with a few remarks:

1. The construction of section 3 can be used to find many new examples of $AdS_3 \times \mathcal{N}$ string backgrounds with spacetime superconformal symmetry. A broad class is given by $\mathcal{N} = \mathrm{U}(1) \times \mathcal{N}_{KS}$ where $\mathcal{N}_{KS}$ is a Kazama-Suzuki model [14] with central charge $c = 9 - 6/k$. Kazama-Suzuki models are gauged $N = 1$ WZW models G/H with an enhanced $N = 2$ worldsheet superconformal symmetry. The cases $(\mathrm{SU}(2)_k \times \mathrm{U}(1)^4)/\mathrm{U}(1)$ or $(\mathrm{SU}(2)_{k'} \times \mathrm{SU}(2)_{k''} \times \mathrm{U}(1))/\mathrm{U}(1)$ are precisely the cases with enhanced spacetime superconformal symmetry discussed above. A simple new case is, for instance, $\mathrm{SU}(3)_{4k}/\mathrm{U}(1)^2$.

2. When the background has an enhanced worldsheet affine algebra, the construction of section 3 can be generalized; in particular, if the enhanced algebra includes an extra affine $\mathrm{U}(1)^2$ factor, $N > 2$ spacetime symmetries can be constructed as in [4].

3. The construction we have given here leads to *conformal* spacetime supersymmetries of the boundary CFT of AdS_3. Other constructions of spacetime supersymmetry are possible, such as the construction with respect to the $\mathrm{U}(1)_R$ of the total worldsheet $N = 2$ superconformal symmetry of $AdS_3 \times \mathcal{N}$ (see, e.g., appendix B of [1]). These in general correspond to different string vacua defined on the same σ-model background, and do *not* give rise to spacetime conformal symmetries[2]. It would be interesting to know if the construction given here is the unique one that does lead to spacetime conformal symmetry (modulo the ambiguity noted in the previous paragraph for spaces with $\mathrm{U}(1)^2$ factors).

[2]Technically, the theories differ because the physical states are required to be mutually local with respect to different spin fields. The spin fields 3.4 are mutually local with respect to the SL(2) currents, whereas the spin fields in the appendix B of [1] are not.

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